Single Pure – Integration By Inspection

All that can come up in Core 3 in integration is the following:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \qquad \int e^x\,dx = e^x + c, \qquad \int \frac{1}{x}\,dx = \ln|x| + c.$$

But with reverse chain rule we also have:

$$\int f'(x)[f(x)]^n\,dx = \frac{[f(x)]^{n+1}}{n+1} + c, \qquad \int f'(x)e^{f(x)}\,dx = e^{f(x)} + c, \qquad \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c.$$

Evaluate the following integrals:

1. $\int (x-1)^3\,dx.$ $\boxed{\frac{(x-1)^4}{4} + c}$
2. $\int 2e^{5x}\,dx.$ $\boxed{\frac{2e^{5x}}{5} + c}$
3. $\int \frac{1}{2x}\,dx.$ $\boxed{\frac{\ln x}{2} + c}$
4. $\int (3x+2)^4\,dx.$ $\boxed{\frac{(3x+2)^5}{15} + c}$
5. $\int -e^{3x-2}\,dx.$ $\boxed{\frac{-e^{3x-2}}{3} + c}$
6. $\int \frac{4}{x-1}\,dx.$ $\boxed{4\ln(x-1) + c}$
7. $\int (5-2x)^{10}\,dx.$ $\boxed{-\frac{(5-2x)^{11}}{22} + c}$
8. $\int \frac{6}{e^{kx}}\,dx.$ $\boxed{-\frac{6}{ke^{kx}} + c}$
9. $\int \frac{-3}{2x+1}\,dx.$ $\boxed{-\frac{3\ln(2x+1)}{2} + c}$
10. $\int \frac{7}{(5-x)^4}\,dx.$ $\boxed{\frac{7}{3(5-x)^3} + c}$
11. $\int \frac{k}{e^{x-1}}\,dx.$ $\boxed{-ke^{1-x} + c}$
12. $\int \frac{7}{1-x}\,dx.$ $\boxed{-7\ln(1-x) + c}$
13. $\int \sqrt{\frac{x}{2} - 3}\,dx.$ $\boxed{\frac{4(\frac{x}{2}-3)^{\frac{3}{2}}}{3} + c}$
14. $\int \frac{e^{2x}}{e^{1+x}}\,dx.$ $\boxed{e^{x-1} + c}$
15. $\int \frac{5}{4x-1} - \frac{6}{7-2x}\,dx.$ $\boxed{\frac{5\ln(4x-1)}{4} + 3\ln(7-2x) + c}$
16. $\int \frac{\pi}{\sqrt{\pi - x}}\,dx.$ $\boxed{-2\pi\sqrt{\pi - x} + c}$
17. $\int xe^{2x^2}\,dx.$ $\boxed{\frac{e^{2x^2}}{4} + c}$
18. $\int \frac{a}{b-kx}\,dx.$ $\boxed{\frac{-a\ln(b-kx)}{k} + c}$
19. $\int x\sqrt{x^2+a}\,dx.$ $\boxed{\frac{(x^2+a)^{\frac{3}{2}}}{3} + c}$
20. $\int 7x^2e^{x^3+1}\,dx.$ $\boxed{\frac{7e^{x^3+1}}{3} + c}$
21. $\int \frac{x}{x^2+a^2}\,dx.$ $\boxed{\frac{\ln(x^2+a^2)}{2} + c}$
22. $\int \frac{x^2}{\sqrt[3]{x^3+1}}\,dx.$ $\boxed{\frac{(x^3+1)^{\frac{2}{3}}}{2} + c}$
23. $\int \frac{\pi e^{x^2+3}}{x^{-1}e^3}\,dx.$ $\boxed{\frac{\pi e^{x^2}}{2} + c}$
24. $\int \frac{e^x}{\pi + e^x}\,dx.$ $\boxed{\ln(\pi + e^x) + c}$
25. $\int \frac{1}{\sqrt[n]{nx+a}}\,dx.$ $\boxed{\frac{(nx+a)^{1-\frac{1}{n}}}{n-1} + c}$
26. $\int e^{\ln x^2} + x\,dx.$ $\boxed{\frac{x^3}{3} + \frac{x^2}{2} + c}$
27. $\int \frac{6x^2+4x}{x^3+x^2}\,dx.$ $\boxed{2\ln(x^3+x^2) + c}$
28. $\int \frac{ax^n}{(x^{n+1}+1)^2}\,dx.$ $\boxed{-\frac{a}{(n+1)(x^{n+1}+1)} + c}$
29. $\int ae^xe^{e^x}\,dx.$ $\boxed{ae^{e^x} + c}$
30. $\int \frac{e^{2x}}{e^{2x}-1}\,dx.$ $\boxed{\frac{\ln(e^{2x}-1)}{2} + c}$
31. $\int \frac{e^x}{\sqrt[n]{e^x+3}}\,dx.$ $\boxed{\frac{n(e^x+3)^{1-\frac{1}{n}}}{n-1} + c}$

32. $\int \frac{e^{\sqrt{x}}}{b\sqrt{x}}\,dx.$ $\boxed{\frac{2e^{\sqrt{x}}}{b}+c}$

33. $\int \frac{ax^3}{x^4+1}\,dx.$ $\boxed{\frac{a\ln(x^4+1)}{4}+c}$

34. $\int \frac{\sqrt{a\ln x+1}}{bx}\,dx.$ $\boxed{\frac{2(a\ln x+1)^{\frac{3}{2}}}{3ab}+c}$

35. $\int \frac{ae^{\frac{1}{x+1}}}{(x+1)^2}\,dx.$ $\boxed{-ae^{\frac{1}{x+1}}+c}$

36. $\int \frac{e^x}{e^{x+1}+k}\,dx.$ $\boxed{\frac{\ln(e^{x+1}+k)}{e}+c}$